Constraint Systems Cheat Sheet

	Boolean Algebra B	Rational Trees RT	Linear Polynomial Equations $\Re$	Finite Domains FD
Domain	Truth values 0 and 1	Herbrand universe with Herbrand interpretation	The set $\Re$ of real numbers	The set $\mathcal{Z}$ of integers

Allowed atomic constraints

Boolean Algebra B

$C ::= X = Y | \neg X = Y \mid X \odot Y = Z$, where X, Y and Z are variables or truth values.

Rational Trees RT

$C ::= s \doteq t$
(s, t: terms over Σ)

Linear Polynomial Equations $\Re$

$C ::= a_1 * X_1 + \ldots + a_n * X_n + b \odot 0, \ \ (n \geq 0)$

- coefficients $a_i, b \in \Re$, $a_i \neq 0$,
- variables $X_1, \ldots, X_n$ totally ordered in strictly descending order
- $\odot \in \{=, <, \leq, >, \geq, \neq\}$

Finite Domains FD

$C ::= X \texttt{ in } n..m \mid X \texttt{ in } [k_1, \ldots, k_l] \mid$

$X \odot Y \mid X + Y = Z$

- n, m, $k_1, \ldots, k_l$: integers ($l \geq 0$)
- $\odot \in \{=, <, >, \leq, \geq, \neq\}$
- X, Y and Z: pairwise distinct variables

Constraint Theory

Boolean Algebra B

Instances of $\neg X = Z$ and $X \odot Y = Z$ according to the following truth table, where $\odot \in \{\sqcap, \sqcup, \oplus, \rightarrow, \leftrightarrow\}$.

X	Y	$\neg X$	$X \sqcap Y$	$X \sqcup Y$
0	0	1	0	0
0	1	1	0	1
1	0	0	0	1
1	1	0	1	1

X	Y	$X \oplus Y$	$X \rightarrow Y$	$X \leftrightarrow Y$
0	0	0	1	1
0	1	1	1	0
1	0	1	0	0
1	1	0	1	1

Rational Trees RT

Clark's Equality Theory Without Acyclity

Reflexivity $\forall(true \rightarrow x \doteq x)$

Symmetry $\forall(x \doteq y \rightarrow y \doteq x)$

Transitivity $\forall(x \doteq y \land y \doteq z \rightarrow x \doteq z)$

Compatibility $\forall(x_1 \doteq y_1 \land \ldots \land x_n \doteq y_n \rightarrow f(x_1, \ldots, x_n) \doteq f(y_1, \ldots, y_n))$

Decomposition $\forall(f(x_1, \ldots, x_n) \doteq f(y_1, \ldots, y_n) \rightarrow x_1 \doteq y_1 \land \ldots \land x_n \doteq y_n)$

Contradiction (Clash) $\forall(f(x_1, \ldots, x_n) \doteq g(y_1, \ldots, y_m) \ \rightarrow \ false)$ if $f \neq g$ or $n \neq m$

Linear Polynomial Equations $\Re$

Tarski's theory of real closed fields

C_1	$(x + y) + z = x + (y + z)$
C_2	$x + 0 = x$
C_3	$x + (-1 * x) = 0$
C_4	$x + y = y + x$
C_5	$(x * y) * z = x * (y * z)$
C_6	$x * 1 = x$
C_7	$x \neq 0 \rightarrow \exists y \ x * y = 1$
C_8	$x * y = y * x$
C_9	$x * (y + z) = (x * y) + (x * z)$
C_{10}	$0 \neq 1$
O_1	$\neg(x < x)$
O_2	$x < y \land y < z \rightarrow x < z$
O_3	$x < y \lor x = y \lor y < x$
O_4	$x < y \rightarrow x + z < y + z$
O_5	$0 < x \land 0 < y \rightarrow 0 < x * y$
R_1	$0 < x \rightarrow \exists y \ y * y = x$
R_2	$y_n \neq 0 \rightarrow \exists x \ y_n * x^n + y_{n-1} * x^{n-1} + \ldots + y_0 = 0$ for odd n

Finite Domains FD

Presburger Arithmetic

$0 = s(X) \rightarrow \bot$

$X = X$

$X = Y \rightarrow s(X) = s(Y)$

$s(X) = s(Y) \rightarrow X = Y$

$X = Y \land Y = Z \rightarrow X = Z$

$X + 0 = X$

$X + s(Y) = s(X + Y)$

$X \leq Y \leftrightarrow \exists Z. X + Z = Y$

$X \texttt{ in } n..m \leftrightarrow n \leq X \land X \leq m$

$X \texttt{ in } [k_1, \ldots, k_l] \leftrightarrow X = k_1 \lor \ldots \lor X = k_l$

CHR rules

Boolean Algebra B

```
a1 @ and(X,Y,Z) <=> X=0 | Z=0.
a2 @ and(X,Y,Z) <=> Y=0 | Z=0.
a3 @ and(X,Y,Z) <=> X=1 | Y=Z.
a4 @ and(X,Y,Z) <=> Y=1 | X=Z.
a5 @ and(X,Y,Z) <=> X=Y | Y=Z.
a6 @ and(X,Y,Z) <=> Z=1 | X=1,Y=1.
```

Analogous for other logical connectives.

Rational Trees RT

```
X eq X <=> var(X) | true.
T eq X <=> var(X),X<<T | X eq T.
T1 eq T2 <=> nonvar(T1),nonvar(T2) |
    same_functor(T1,T2),
    args2list(T1,L1),args2list(T2,L2),
    same_args(L1,L2).
X eq T1, X eq T2 <=>
  var(X),X<<T1,T1=<<T2 |
    X eq T1, T1 eq T2.
```

Linear Polynomial Equations $\Re$

```
eliminate @ A1*X+P1 eq 0 \ PX eq 0 <=>
    find(A2*X,PX,P2) |
    normalize(A2*(-P1/A1)+P2,P3),
    P3 eq 0.
empty @ B eq 0 <=> number(B) |
    zero(B).
```

Finite Domains FD

```
X in A..B <=> A>B | false.
X in A..B, X in C..D <=>
    X in max(A,C)..min(B,D).
X le Y, X in A..B, Y in C..D <=> B>D |
    X le Y, X in A..D, Y in C..D.
X le Y, X in A..B, Y in C..D <=> C<A |
    X le Y, X in A..B, Y in A..D.
X eq Y, X in A..B, Y in C..D <=> A\=C |
    X eq Y,
    X in max(A,C)..B, Y in max(C,A)..D.
X eq Y, X in A..B, Y in C..D <=> B\=D |
    X eq Y,
    X in A..min(B,D), Y in C..min(D,B).
X ne Y, X in A..B, Y in C..D <=> A=C,C=D |
    X ne Y, X in (A+1)..B, Y in C..D.
```