Constraint Programming Cheat Sheet

	Constraint Logic Programming	Concurrent Constraint Logic Programming	Constraint Handling Rules
Syntax	*Atom*: $A, B ::= p(t_1, \ldots, t_n),\ n \geq 0$ *Constraint*: $C, D ::= c(t_1, \ldots, t_n) \ \mid C \wedge D, n \geq 0$ *Goal*: $G, H \ ::= \ \top \mid \bot \mid A \mid C \mid G \wedge H$ *CL Clause*: $K ::= A \leftarrow G$ *CL Program*: $P ::= K_1 \ldots K_m,\ m \geq 0$	*Atom*: $A, B ::= p(t_1, \ldots, t_n),\ n \geq 0$ *Constraint*: $C, D ::= c(t_1, \ldots, t_n) \ \mid C \wedge D, n \geq 0$ *Goal*: $G, H ::= \top \mid \bot \mid A \mid C \mid G \wedge H$ *CCL Clause*: $K ::= A \leftarrow C \mid G$ *CCL Program*: $P ::= K_1 \ldots K_m,\ m \geq 0$	*CHR Constraint*: $E, F ::= e(t_1, \ldots, t_n) \mid E \wedge F, n \geq 0$ *Built-in Constraint*: $C, D ::= c(t_1, \ldots, t_n) \mid C \wedge D, n \geq 0$ *Goal*: $G, H ::= \top \mid \bot \mid C \mid E \mid G \wedge H$ *CHR Rule*: $R ::= E \Leftrightarrow C \mid G \mid E \Rightarrow C \mid G$ *CHR Program*: $P ::= R_1 \ldots R_m, m \geq 0$
Operational Semantics	**Unfold** *If* $(B \leftarrow H)$ is a fresh variant of a clause in P *and* $\mathcal{CT} \models \exists\, ((B \doteq A) \wedge C)$ *then* $\langle A \wedge G, C\rangle \mapsto \langle H \wedge G, (B \doteq A) \wedge C\rangle$ **Failure** *If* there is no clause $(B \leftarrow H)$ in P *with* $\mathcal{CT} \models \exists\, ((B \doteq A) \wedge C)$ *then* $\langle A \wedge G, C\rangle \mapsto \langle \bot, \mathit{false}\rangle$ **Solve** *If* $\mathcal{CT} \models \forall\, ((C \wedge D_1) \leftrightarrow D_2)$ *then* $\langle C \wedge G, D_1\rangle \mapsto \langle G, D_2\rangle$	**Unfold** *If* $(B \leftarrow D \mid H)$ is a fresh variant of a clause in P with variables $\bar{X}$ *and* $\mathcal{CT} \models \forall\, (C \rightarrow \exists \bar{X}((B \doteq A) \wedge D))$ *then* $\langle A \wedge G, C\rangle \mapsto$ $\langle H \wedge G, (B \doteq A) \wedge D \wedge C\rangle$ **Solve** *If* $\mathcal{CT} \models \forall\, ((C \wedge D_1) \leftrightarrow D_2)$ *then* $\langle C \wedge G, D_1\rangle \mapsto \langle G, D_2\rangle$	**Simplify** *If* $(F \Leftrightarrow D \mid H)$ is a fresh variant of a rule in P with variables $\bar{X}$ *and* $\mathcal{CT} \models \forall (C \rightarrow \exists \bar{X}(F \doteq E \wedge D))$ *then* $\langle E \wedge G, C\rangle \mapsto \langle H \wedge G, (F \doteq E) \wedge D \wedge C\rangle$ **Propagate** *If* $(F \Rightarrow D \mid H)$ is a fresh variant of a rule in P with variables $\bar{X}$ *and* $\mathcal{CT} \models \forall (C \rightarrow \exists \bar{X}(F \doteq E \wedge D))$ *then* $\langle E \wedge G, C\rangle \mapsto \langle E \wedge H \wedge G, (F \doteq E) \wedge D \wedge C\rangle$ **Solve** *If* $\mathcal{CT} \models (C \wedge D_1) \leftrightarrow D_2$ *then* $\langle C \wedge G, D_1\rangle \mapsto \langle G, D_2\rangle$
Declarative Semantics	*Main definition $P^{\leftrightarrow} \cup \mathcal{CT}$ **(1) Completion $P^{\leftrightarrow}$** For each p/n in P add to $P^{\leftrightarrow}$ the formula $p(\bar{t}_1) \leftarrow G_1 \quad \ldots \quad p(\bar{t}_m) \leftarrow G_m$ $\forall \bar{X}(p(\bar{X}) \leftrightarrow \exists \bar{Y}_1(\bar{t}_1 \doteq \bar{X} \wedge G_1) \vee \ldots \vee \exists \bar{Y}_m(\bar{t}_m \doteq \bar{X} \wedge G_m))$ *G_i includes the guards C_i for CCLP	**(2) $\mathcal{CT}$ including Clark's Equality Theory (CET)** *Reflexivity* $(\top \rightarrow X \doteq X)$ *Symmetry* $(X \doteq Y \rightarrow Y \doteq X)$ *Transitivity* $(X \doteq Y \wedge Y \doteq Z \rightarrow X \doteq Z)$ *Compatibility* $(X_1 \doteq Y_1 \wedge \ldots \wedge X_n \doteq Y_n$ $\rightarrow f(X_1, \ldots, X_n) \doteq f(Y_1, \ldots, Y_n))$ *Decomposition* $(f(X_1, \ldots, X_n) \doteq f(Y_1, \ldots, Y_n)$ $\rightarrow X_1 \doteq Y_1 \wedge \ldots \wedge X_n \doteq Y_n)$ *Contradiction (Clash)* $(f(X_1, \ldots, X_n) \doteq g(Y_1, \ldots, Y_m) \rightarrow \bot)$ if $f \neq g$ or $n \neq m$ *Acyclicity* $(X \doteq t \rightarrow \bot)$ if t is function term and X appears in t	CHR-Rule — Logical Reading **Simplify** $E \Leftrightarrow C \mid G$ — $\forall \bar{X}(C \rightarrow (E \leftrightarrow \exists \bar{Y}\ G))$ **Propagate** $E \Rightarrow C \mid G$ — $\forall \bar{X}\ (C \rightarrow (E \rightarrow \exists \bar{Y}\ G))$
Soundness	If a goal G has successful derivation with answer constraint C, then $P^{\leftrightarrow} \cup \mathcal{CT} \models \forall (C \rightarrow G)$.	If a goal G has successful derivation with answer constraint C, then $P^{\leftrightarrow} \cup \mathcal{CT} \models \forall (C \rightarrow G)$.	If a goal G has a derivation with answer constraint C then $\mathcal{P} \cup \mathcal{CT} \models \forall\ (C \leftrightarrow G)$.
Completeness	If $P^{\leftrightarrow} \cup \mathcal{CT} \models \forall (C \rightarrow G)$ and C is satisfiable in $\mathcal{CT}$, then there are successful derivations for G with answer constraints $C_1, \ldots, C_n$ s.t. $\mathcal{CT} \models \forall (C \rightarrow (C_1 \vee \ldots \vee C_n))$.	If P is a deterministic CCL program, G is a goal with at least one fair derivation, $P^{\leftrightarrow} \cup \mathcal{CT} \models \forall (C \rightarrow G)$ and C is consistent in $\mathcal{CT}$ then each successful derivation of G has an answer constraint C' s.t. $\mathcal{CT} \models \forall (C \rightarrow C')$.	If G with at least one finite derivation $\mathcal{P} \cup \mathcal{CT} \models \forall\ (C \leftrightarrow G)$ then G has a derivation with answer constraint C' s.t. $\mathcal{P} \cup \mathcal{CT} \models \forall\ (C \leftrightarrow C')$.
Soundness and Completeness of Failed Derivations	Let P be a CL program and G a goal. $P^{\leftrightarrow} \cup\ \mathcal{CT} \models \neg\exists G$ if and only if each fair derivation starting with $\langle G, \mathit{true}\rangle$ fails finitely.	Let P be a deterministic CCL program and G a goal with at least one fair derivation, then the following statements are equivalent: $P \leftrightarrow \cup\ \mathcal{CT} \models \neg\exists G$; G has a finitely failed derivation; and each fair derivation of G fails finitely.	If P is terminating and confluent CHR program, and G is a goal with at least one answer constraint consisting of only built-in constraints then $P \cup \mathcal{CT} \models \neg\exists G$ if and only if each finite derivation starting with $\langle G, \mathit{true}\rangle$ fails.