Constraint Programming

Prof. Dr. Thom Frühwirth
Amira Zaki

Winter Term 2012
Assignment #1

Logic Refresher Exercises

Exercise 1. Check for each of the following formulae which properties are true: validity, satisfiability, falsifiability, unsatisfiability.

(1) $A \vee \neg A$
(2) $A \wedge \neg A$
(3) $A \rightarrow \neg A$
(4) $A \rightarrow (B \rightarrow A)$
(5) $A \rightarrow (A \rightarrow B)$
(6) $A \leftrightarrow \neg A$
(7) $(A \wedge B) \rightarrow (A \vee B)$
(8) $(A \vee B) \rightarrow A$
(9) $(A \wedge B) \wedge \neg A$

Exercise 2. For a formula F, are the following statements true or false? Give a counter example for each false statement.

(1) If F is valid, then F is satisfiable.
(2) If F is satisfiable, then $\neg F$ is unsatisfiable.
(3) If F is valid, then $\neg F$ is unsatisfiable.
(4) If F is unsatisfiable, then $\neg F$ is valid.

Exercise 3. Is G a logical consequence of F? For each state if $F \models G$ or $F \not\models G$.

F	G	$F \models G$ or $F \not\models G$
A	$A \vee B$	
A	$A \wedge B$	
A, B	$A \vee B$	
A, B	$A \wedge B$	
$A \wedge B$	A	
$A \vee B$	A	
$A, (A \rightarrow B)$	B	

Exercise 4. For each pair of atomic formulae, give both a most general unifier and a unifier which is not most general. Otherwise show that a unifier cannot exist.

(1) $p(a, X, c, Y, Z)$ and $p(Y, X, c, a, W)$
(2) $p(Y, g(Y, f(Y)), g(X, a))$ and $p(f(U), V, g(h(U, V, W), U))$
(3) $p(g(X), h(a, X), h(Y, Y))$ and $p(U, h(a, g(V)), h(V, g(U)))$
(4) $p(h(f(X), a), X, h(f(f(X)), f(f(X))))$ and $p(h(V, a), U, h(W, f(V)))$

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Assignment #2

Introducing SWI-Prolog

Exercise 1. Hello World

- Download SWI-Prolog to your personal machine. It is already installed in the PC pool.
- Go through the SWI-Prolog reference manual:
  http://www.swi-prolog.org/pldoc/refman/
- Write a "Hello world!" program in Prolog, e.g.,
  `start :- write('Hello world!').`
- Consult the source file and call it with the goal `start.`

Exercise 2. Family Relations

- Create the family database adding facts, representing the family tree as follows:
  (1) `male(X)` : `X` is a male
  (2) `female(X)` : `X` is a female
  (3) `parent(X,Y)` : `X` is a parent of `Y`
- Implement predicates to represent the following relationships:
  (1) `mother(X,Y)`
  (2) `father(X,Y)`
  (3) `grandmother(X,Y)`
  (4) `grandfather(X,Y)`
  (5) `ancestor(X,Y)`
  (6) `brother(X,Y)`
  (7) `sister(X,Y)`
  (8) `siblings(X,Y)`
  (9) `uncle(X,Y)`
  (10) `cousin(X,Y)`
- Add facts about your own family and test if your predicates work correctly.

Exercise 3. Unification

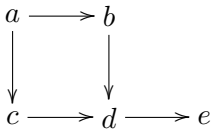
First attempt this exercise by hand without the help of SWI-Prolog, then verify your solutions by testing them. Specify whether the query submitted to Prolog succeeds or fails; and if it succeeds, specify what is assigned to the variables by the unification; if it fails, explain why.

(1) `?- X = hans.`
(2) `?- anja = hans.`
(3) `?- X = hans, X = Y.`
(4) `?- X = happy(hans).`
(5) `?- X is Y.`
(6) `?- 2 + 1 = 3.`
(7) `?- f(X,a) = f(a,X).`
(8) `?- likes(anja, X) = likes(X, hans).`
(9) `?- A = b(c).`
(10) `?- a = b(c).`

The next exercise to be submitted by e-mail to: amira.zaki@uni-ulm.de. The deadline is on 07.11.2012 by 10:00. You are allowed to work in a group of two people. Please send only one e-mail per group, containing the solution and both team member names.

Exercise 4. Directed Graphs

A directed graph can be represented by storing information about the edges contained as facts. Model the given directed graph into respective facts.



Implement a predicate `path(X,Y)` that holds if there exists a path from the node `X` to the node `Y`. Try the execution of the following goals and trace their execution:

`path(a,c).` `path(b,Y).` `path(f,g).`

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Assignment #3

To follow the execution of Prolog programs use the *tracer*. To enable the tracer, enter "`trace`". To disable it again, use "`notrace`". The tracer will show the computation step by step. A debugger is also possible, enabled by "`debug`". Check the *SWI-Prolog manual* for more options.

Exercise 1. Write the following arithmetic Prolog predicates:

(1) `minimum(A,B,C)` which succeeds if `C` is the minimum of `A` and `B`.
(2) `factorial(X,N)` which succeeds if `N` is the factorial of `X`.

Exercise 2. Write the following list manipulating Prolog predicates:

(1) `member(X,L)` which succeeds if `X` is a member of the list `L`.
(2) `len(N,L)` which succeeds if `N` is the length of a list `L`.
(3) `append(L1, L2, L3)` which succeeds if `L3` is the concatenation of `L1` and `L2`.
(4) `reverse(L1,L2)` which succeeds if the list `L2` is the reverse of the list `L1`.
(5) `remove(X,L1, L2)` which succeeds if `L2` is the result of removing `X` from `L1`.
(6) `union(L1, L2, L3)` which succeeds if `L3` is the union of `L1` and `L2`.
(7) `intersect(L1, L2, L3)` which succeeds if `L3` is the intersection of `L1` and `L2`.

The next exercise to be submitted by e-mail to: amira.zaki@uni-ulm.de. The deadline is on 21.11.2012 by 10:00. You are allowed to work in a group of two people. Please send only one e-mail per group, containing the solution and both team member names.

Exercise 3. Implement a Prolog program to help scheduling routes for the freight forwarding business. The program uses the following knowledge base, giving distances between major cities in Germany.

```
Hamburg, Bremen, 80          Hamburg, Hannover,110
Hamburg, Berlin, 230         Bremen, Hannover,100
Bremen, Dortmund, 200        Hannover, Kassel,140
Hannover, Nuernberg, 380     Dortmund, Kassel,130
Dortmund, Koeln, 80          Kassel, Wuerzburg,180
Kassel, Frankfurt, 180       Frankfurt, Wuerzburg,110
Nuernberg, Muenchen, 160
```

a) Represent the distances with a Prolog predicate `dist(city0,city1,km)`. Test your implementation with queries like `?- dist(dortmund,bremen,Km)`, `?- dist(From,To,180)`, and `?- dist(From,To,Km)`.

b) The Prolog predicate `route(From,To)` should succeed, iff there is a route between `From` and `To`. Tests should include `route(muenchen,berlin)`, `route(ulm,halle)`, and finally `route(frankfurt,To)`.

c) Define a ternary predicate `route(From,To,Route)` which has the same functionality as `route/2` w.r.t. the first and second parameters `From` and `To`. Additionally, the third parameter, `Route`, contains the cities already visited in order to avoid cycles (i.e. visiting a city more than once). Testing should include ground and non-ground queries, e.g., `?- route(dortmund,X,[kassel])`.

d) The Prolog predicate `shortestpath(From,To,Route,Km)` should succeed, iff the minimal distance between the cities `From` and `To` is `Km` along `Route`. Tests should include, e.g., `shortestpath(kassel,bremen,[hannover],X)`.

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Assignment #4

Constraint Logic Programming

The SWI-prolog documentation of CLP-FD library:
http://www.swi-prolog.org/man/clpfd.html

Exercise 1 (Cryptoarithmetic Puzzle)**.**
Replace distinct letters by distinct digits (numbers have no leading zeros), such that the following calculation holds (a literal translation to English is "Test thoroughly your strengths"):

		T	E	S	T	E
+		F	E	S	T	E
+		D	E	I	N	E
=	K	R	Ä	F	T	E

Use the `clpfd` library to model the previous puzzle as done in the lecture. You might find it useful to use the constraint `all_different(+Vars)`.

Exercise 2. Model the following dinner problem as a constraint problem in `clpfd`:
We are going out to dinner taking 1-6 grandparents, 1-10 parents and 1-40 children. Grandparents cost 3 euros for dinner, parents cost 2 euros and children 0.50 euros. There must be 20 total people at dinner and it must cost exactly 20 euros. The problem to be solved is to find how many grandparents, parents and children are going to dinner.

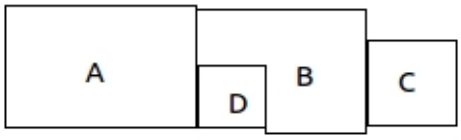
Exercise 3. Can you find the ages according to the following dialogue?

```
Alex: How old are you mama?
Mama: Our three ages add up to exactly seventy years.
Alex: And how old are you papa?
Papa: Just six times as old as you, my son.
Alex: Shall I ever be half as old as you, papa?
Papa: Yes Alex; and when that happens our three ages will
      add up to exactly twice as much as today.
```

Write a `clpfd` program to solve this puzzle and return the age of Alex, Mama and Papa today.
Hint: model the age in months.

Exercise 4. The map coloring problem tries to find a way to color a map such that no two boarding states have the same color. The predicate `color(States, NumColors, Adj)` should succeed iff it is possible using a maximum number of colors, `NumColors`, and the list of adjcacent pair of states, `Adj`, to satifty the map coloring rules and color the states as in `States`.

Implement the predicate `color` using `clpfd`. Test coloring of the 4 states shown on the right using 3 colors, with the predicate:




```
?- color([A,B,C,D],3,[(A,B),(B,C),(B,D),(D,A)]).
```

Exercise 5 (Magic Square)**.**
A "magic square" is a rectangular array of *distinct* numbers, usually from 1 to n^2, such that each column, row, and both diagonals have the same sum. The constant sum in every row, column and diagonal is called the magic sum, `M`. Using the `clpfd` library, solve the puzzle for a magic square with $n = 4$ and $M = 34$. Make sure you write the full square on the console.

The next exercise is to be submitted by e-mail to: amira.zaki@uni-ulm.de. The deadline is on 27.11.2012 by 12:00. You are allowed to work in a group of two people. Please send only one e-mail per group, containing the solution and both team member names.

Exercise 6 (Survo Puzzle)**.**
Survo puzzle is a kind of logic puzzle presented (in April 2006) and studied by Seppo Mustonen. The name of the puzzle is associated to Mustonen's Survo system which is a general environment for statistical computing and related areas.

In a Survo puzzle the task is to fill an $m \times n$ table by integers $1, 2, ..., m \times n$ so that each of these numbers appears only once and their row and column sums are equal to integers given on the bottom and the right side of the table. Often some of the integers are given readily in the table in order to guarantee uniqueness of the solution and/or for making the task easier [taken from Wikipedia].

Here is a simple Survo puzzle with 3 rows and 4 columns:

	A	B	C	D	
1		6			30
2	8				18
3			3		30
	27	16	10	25	

Solve the Survo puzzle using the `clpfd` library for 3×4 tables.
Extra: Think of a generic solution for any Survo puzzle.

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Assignment #5

Write and query Prolog, CLP and CHR online using:
http://chr.informatik.uni-ulm.de/~webchr/

CCLP

We use (a subset of) the Constraint Handling Rules (CHRs) to program in the CCLP paradima.
The following CCLP-clause $H \leftarrow C \mid G$
is written as `N @ H <=> C | G`
where `N @` is an optional name for the rule.

- Read the SWI-Prolog manual on CHR: http://www.swi-prolog.org/man/chr.html
- Before using CHR, the CHR library must be included, `:- use_module(library(chr)).`
- User-defined CCLP predicates must be declared, `:- chr_constraint contraint/arity.`
- To enforce that guards should be checked for (illegal) variable bindings, use, `:- chr_option(check_guard_bindings,on).`

Exercise 1 (Comparison of CLP and CCLP)**.**
Compare the following CLP- (in the left column) and CCLP-programs (in the right column), which consist of *one* of the given rules by posing the queries given below. Check your answers with the system's answers. Make sure, you understand why seemingly innocuous rules produce different answers.

```
p(a) :- true.
p(X) :- X=a.
p(X) :- X = a, X = b.
```

```
p1 @ p(a) <=> true | true.
p2 @ p(X) <=> X = a | true.
p3 @ p(X) <=> true | X = a.
p4 @ p(X) <=> X = a, X = b | true.
p5 @ p(X) <=> X = a | X = b.
p6 @ p(X) <=> true | X = a, X = b.
```

Queries: (a) `p(a)`, (b) `p(b)`, and (c) `p(C)`.

Exercise 2. Implement the following three variants of the CCLP minimum program in CHR:

- Variant 1:

$$min1_1(X,Y,Z) \leftarrow \top \mid X \leq Y,\ Z = X$$
$$min1_2(X,Y,Z) \leftarrow \top \mid Y \leq X,\ Z = Y$$

- Variant 2:

$$min2_1(X,Y,Z) \leftarrow X \leq Y \mid Z = X$$
$$min2_2(X,Y,Z) \leftarrow Y \leq X \mid Z = Y$$

- Variant 3:

$$min3_1(X,Y,Z) \leftarrow X \leq Y,\ Z = X \mid \top$$
$$min3_2(X,Y,Z) \leftarrow Y \leq X,\ Z = Y \mid \top$$

Test and explain the different responses of the variants by posing the following six queries (only one at a time):

```
min(1,2,C).   min(A,2,1).   min(A,2,3).
min(A,A,B).   min(1,2,1).   min(1,2,3).
```

Exercise 3 (Hamming's problem)**.**
Consider the classical *Hamming's problem*, which is to compute an ordered ascending chain of all numbers whose only prime factors are $2, 3$, or 5. The chain starts with the numbers:

`1,2,3,4,5,6,8,9,10,12,15,16,18,20,24,25,...`

Define a non-terminating process `hamming(N)` that will produce the numbers as elements of the infinite chain starting with 1. *Hints:*

- pretend that the sequence is already known
- no base cases in recursion as sequences are infinite
- concurrent-process network, processes can be executed in parallel
- to view the stream, you will need an `observe/1` rule that writes elements of the stream as they are produced

Exercise 4 (Bank Transactions)**.**
A classical and good example of a producer/consumer interaction is a bank account model.
The bank "consumes" client's commands which are in the form:

`withdraw(Amount)` - withdraws the `Amount` from the client's balance
`deposit(Amount)` - deposits the `Amount` to the client's balance
`balance(Amount)` - binds `Amount` to the client's balance

where `Amount` is a free variable.
For example, a client could produce the following:

$$[deposit(100), withdraw(25), balance(75)]$$

The bank process observes these commands and in its second argument modifies the current balance accordingly. Implement CHR `bank/2` rules that handle a stream of the three possible client commands and update the balance accordingly. Also implement a starter rule `bank/1` which initially sets the balance to zero, then calls the `bank/2` handlers.

You can use the Prolog predicate `read/1` to read the next Prolog term from the current input stream. Thus additionally, implement a producer/client (`client/1`) that would continuously read a term from the user.
The idea is to create these consumer and producer "threads", to run them using the query:

```
?- bank(L),client(L).
```

The next exercises are to be submitted by e-mail to: amira.zaki@uni-ulm.de. The deadline is on 05.12.2012 by 10:00. You are allowed to work in a group of two people. Please send only one e-mail per group, containing the solution and both team member names.

Exercise 5 (Fibonacci Sequence)**.**
By definition, the first two numbers in the Fibonacci sequence are 0 and 1, and each subsequent number is the sum of the previous two. The chain starts with the numbers:

`0,1,1,2,3,5,8,13,21,34,55,89,144,...`

Define a non-terminating process `fib(N)` that will produce the numbers as elements of the infinite chain starting with 0. *Hint:* You want to create an infinite stream, so do not create a base case and you can provide the first two Fibonacci elements in the query, along with the `observe/1`.

Exercise 6 (Minimum of Stream)**.** Create a recursive stream that produces minimal elements found so far in a stream (i.e. a sequence of current minima) using `min/3`. An example query:

```
?- observe(ML), ML=[31|_], minlist([31,56,13,45,24,52,2,634,58,632,23,1543],ML).
```

would produce the stream: `31,31,31,13,13,13,13,2,2,2,2,2,2`.

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Assignment #6

Constraint Handling Rules

For debugging, you can use the tracer. To enable the tracer, enter "`chr_trace`". To disable it, use "`chr notrace`". The tracer will show you the computation step by step.

Exercise 1. Compare the following CHR programs, which consist of *one* of the given rules by posing the given queries. Check your answers with the system's answers. Make sure you understand why seemingly similar rules produce different answers.

```
c1 @ c(X), c(X) <=> q(X,X).
c2 @ c(X), c(Y) <=> r(X,Y).
c3 @ c(X), c(X) ==> q(X,X).
c4 @ c(X), c(Y) ==> r(X,Y).
```

Queries:

a) `c(X), c(X)`
b) `c(X), c(Y)`
c) `c(X), c(Y), X=Y`

More variants:

```
q1 @ p(X,Z), q(Z,Y) <=> q(X,Y).
q2 @ q(Z,Y), p(X,Z) <=> q(X,Y).
q3 @ p(X,Z), q(Z,Y) ==> q(X,Y).
q4 @ q(Z,Y), p(X,Z) ==> q(X,Y).
q5 @ p(X,Z) \ q(Z,Y) <=> q(X,Y).
q6 @ q(Z,Y) \ p(X,Z) <=> q(X,Y).
```

Queries:

d) `p(a,b), q(b,c)`
e) `p(a,b), q(b,c), p(d,a)`

Comment on the system's answers for queries a) to e). Comment on the system's answers for the rule `q5` and the following two queries:

`p(X,C), p(Y,C), q(C,A)` and `p(Y,C), p(X,C), q(C,A)`.

Exercise 2. Implement the constraints `less/2` (encoding $<$) und `leq/2` (encoding $\leq$) and their mutual relations/interactions in CHR. Use the lecture's CHR program for the $\leq$ constraint helpful (also present on WebCHR). For an example query, take your last name as a sequence of variables with $\leq$ constraints between succeeding characters. The name *Fruehwirth* translates to the query:
`leq(F,R),leq(R,U),leq(U,E),leq(E,H),leq(H,W),leq(W,I),leq(I,R),leq(R,T),leq(T,H)`
with answer `leq(F,E), R = T, U = T, E = T, H = T, W = T, I = T` .
Do additional tests consisting of combined `less` and `leq` constraints.

Submit the next exercise by e-mail to: amira.zaki@uni-ulm.de latest by 19.12.2012 by 10:00.

Exercise 3. The addition of natural numbers written in successor notation (the natural number 3 is written as `s(s(s(0)))`) can be implemented by:

```
add(0,X,X).
add(s(X),Y,s(Z)) :- add(X,Y,Z).
```

Write a constraint solver in CHR, that solves as many as possible single `add`-constraints, e.g., the first Prolog clause implies:

```
add(0,Y,Z) <=> Y=Z.
add(X,Y,Y) <=> X=0.
```

You will need to write the other base cases and the recursive cases required for the addition.

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Assignment #7

Exercise 1 (Warmup – generate and test)**.**
Implement a Prolog-predicate `permutation/2` that generates permutations: `permutation(A,B)` generates a permutation `B` of a list `A` with fixed length. All permutations can be computed by backtracking using ";" when promted. Do not use built-in predicates for manipulating lists. Describe your observations with the following queries:

- `permutation([1,2,3],[L,M,N]).`
- `permutation([L,M,N],[1,2,3]).`
- `permutation([1,2,3],B).`
- `permutation(A,[1,2,3]).`

Exercise 2 (Permutation Sort – "generate and test")**.**
Use the *permutation sort* algorithm for sorting a list of integers. To implement the *generate and test version*, use three Prolog predicates `permsort(List,Sorted)`, `permutation(List,Sorted)`, and `sorted(Sorted)` (all arguments are lists).

Exercise 3 (Permutation Sort – "constrain and generate")**.** Use the CHR Constraint `leq/2` from assignment #6-2 for a "constrain-and-generate" version of the *permutation sort* algorithm, replacing the Constraint `=<` by the CHR-constraint `leq`.
Your tests should (at least) include the following queries

```
?- permsortCHR([1,A,3],[1,3,7]).
?- permsortCHR([2,A],X).
?- permsortCHR([A,B,A],X).
?- permsortCHR(List,[1,X,3]).
?- permsortCHR([1,X,Y],[X,1,Y]), permsortCHR([4,5,10], [Z,Y,W]).
```

Submit the next exercise by e-mail to: amira.zaki@uni-ulm.de, latest by 09.01.2013 at 10:00.

Exercise 4. Write a CHR program for the `maximum(X,Y,Z)`-constraint, which succeeds iff `Z` is the maximum of `X` and `Y`. Use your implementation of the `leq/2`- and `less/2`-constraints from assignment #6-2. Consider the following items in this order:

(1) Write a CHR rule, which computes the maximum `Z` of two given numbers `X` and `Y`.
(2) Enhance by inserting a CHR rule, such that queries like `?- maximum(X,X,3)` can be handled satisfactorily (we expect `X = 3`).
(3) Insert a CHR rule which, given the constraint `maximum(X,Y,Z)`, propagates the constraints `leq(X,Z)` and `leq(Y,Z)`. Test with the query `?- maximum(A,B,C),maximum(C,A,B).`
(4) Insert CHR rules, such that under a given unequality between `X` and `Y` (e.g. `X leq Y`) the constraint `maximum(X,Y,Z)` is replaced by the implied equality (e.g. `Y = Z`).
(5) Insert a CHR rule, such that for a query like `?- less(X,Z), maximum(X,Y,Z)` the answer `Y=Z` is returned.
(6) In order to handle queries like `?- maximum(X,Y,3),maximum(X,Y,5)` in a satisfactory manner, extend the program for a rule for this class of queries.
(7) Which are the sufficient conditions, given that two variables are unequal, to allow more inferences from `maximum(X,Y,Z)`.
Implement this case by (additional) rules in your CHR program.

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Assignment #8

Constraint System *B*

Download `boole.pl` from the lecture web page (also sent to your email): http://www.uni-ulm.de/fileadmin/website_uni_ulm/iui.inst.170/home/betz/boole.pl. It contains the implementation of the constraints `neg/2`, `and/3`, `or/3`, `xor/3`, and `imp/2` of the Boolean Algebra. Use this constraint-solver for the following exercises.

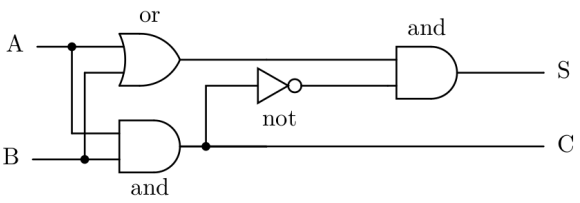
Exercise 1 (Equivalence)**.**

Extend `boole.pl` with rules (similar to the ones already defined) in order to introduce equivalence, i.e. implement simplifications for a CHR-constraint `equiv(X,Y,Z)` which obey the given truth table.

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

Exercise 2 (Half Adder)**.**

Write a CHR constraint `add(A,B,S,C)`, which implements a half adder by means of Boolean constraints.
Test with queries `add(1,0,S,C)`, `add(A,B,S,1)` and `add(A,B,S,0)`.



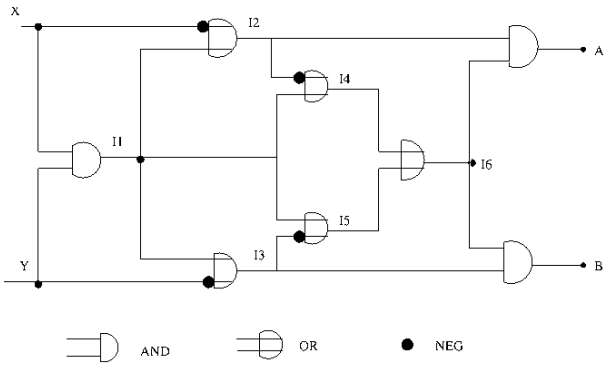
Exercise 3 (Who is lying?)**.**

Lehmann says Mueller lies. Mueller says Schulze does not tell the truth. Schulze says both lie.

Write a Prolog-predicate `tellTruth(Lehmann,Mueller,Schulze)` which determines who of the three people is lying and who is telling the truth; it should succeed iff the three arguments are a valid interpretation of the given statements by Lehmann, Mueller, and Schulze. Use the Boolean constraints `and`, `neg`, ...
Hint: Lehmann's statement can be modeled by `Lehmann = MuellerLies`, or using equivalence, with `MuellerLies` being the negation of `Mueller`.

Exercise 4 (Cross Circuit).

A cross circuit exchanges two wires/signals with the help of a logic circuit without crossing them physically. For the input pins (X, Y) and the output pins (A, B) we have $A = Y$ and $B = X$.
Write a CHR constraint `cross(X,Y,A,B)`, which implements a cross circuit by means of Boolean constraints.
Test with queries `cross(1,0,A,B)`, `cross(1,Y,1,B)` and `cross(0,Y,A,B)`.



Submit the next exercises by e-mail to: amira.zaki@uni-ulm.de, latest by 23.01.2013 at 10:00.

Exercise 5 (To like or not to like).
I know three people: Pat, Quincy and Ray. I like at least one of them. If I like Pat but not Ray then I also like Quincy. I like both Ray and Quincy or none of them. Describe the facts as Boolean constraints and determine who I like for sure and who I may dislike.

Exercise 6 (De Morgan's laws).
Translate the rewrite rules for De Morgan's law of propositional logic into CHR simplification rules:

```
not(and(X,Y)) ->  or(not(X),not(Y)).
not(or(X,Y))  ->  and(not(X),not(Y)).
```

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Amira Zaki

Winter Term 2012
Assignment #9

Constraint System *FD*

Exercise 1 (Extension of *FD*-solver)**.**
Extend the finite domain constraint solver over **enumeration domains** presented in the lecture.

a) Implement rules to let the `leq`, `eq`, and `neq` constraints interact with the given solver. Tests should include `X leq Y, Y leq Z, X in [0,3], Y in [-1,2], Z in [0,1,2,3]`.
b) Introduce the `maximum(X,Y,Z)` constraint to the given solver, where `Z` is the maximum of `X` and `Y`. Tests should include `maximum(X,Y,Z),X in [0,1],Y in [2,4],Z in [3,4]`.
c) Implement a `label(List)`-constraint to bind the variables in `List`. An easy test case is, e.g. `X in [0,1], Y in [0,1], X leq Y, label([X])`.

Exercise 2 (N-Queens)**.**
Implement a solver for the N-Queens puzzle as covered in the lecture. You can change `no_attack/3` to the following:

```
no_attack(X,Y,N) <=> X neq Y,
                     add(X,N,XPN), XPN neq Y,
                     add(Y,N,YPN), YPN neq X.
```

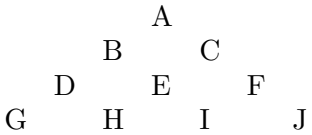
Hint: You will need to implement a special handler for the `add/3` *constraint. Only consider propagation where the first input values is fixed/known and the second is a number.*

Exercise 3 (Sudoko)**.**
Sudoko is a logic puzzle which is solved using logic and reasoning. The objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids (or blocks) that compose the grid contains all of the digits from 1 to 9. Implement a sudoko solver in CHR using the above FD-solver.

Hint: Represent each puzzle cell using a constraint `cell(R,C,B,V)`*, where the value* `V` *is stored in the puzzle at row* `R` *and column* `C` *which belongs to the sub-grid or block* `B`*. Then write the appropriate rules that propagate the relevant* `neq` *constraints. Create a* `sudoko` *constraint that produces all 64 cell constraints with the correct variables.*

Submit the next exercise by e-mail to: amira.zaki@uni-ulm.de, latest by 30.01.2013 at 10:00.

Exercise 4 (Pyramid – Crypto-arithmetic Puzzle)**.**
Solve the following crypto-arithmetic puzzle using finite domain constraints. Replace distinct letters by distinct digits, such that each number is the absolute difference of the two numbers below (e.g. $A = |B - C|$), and the numbers are the positive integers from 1 to 10.



Hint: You will need to implement CHR rules for an absolute difference `abs/3` *constraint, only consider propagation where one of the input values is fixed/known. Additionally implement a constraint* `allneq(List)` *that succeeds iff the variables in* `List` *are (pairwise) distinct.*

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Assignment #10

Constraint System *Rational Trees*

Exercise 1. Implement the axioms of the theory of rational trees for the CHR equality constraint `X eq Y`.

Hints:

- Change terms to list of functor and arguments by: `f(X1,...,XN)=..[f,X1,...,XN]`
- Rules leading to immediate contradiction should go first in the program text
- For termination reasons remove duplicate constraints

Queries:

(a) `p(a,X,c,Y,Z) eq p(Y,X,c,a,W)`
gives $Y = a$, $Z = W$

(b) `p(Y,g(Y,f(Y)),g(X,a)) eq p(f(U),V,g(h(U,V,W),U))`
gives $Y = f(a)$, $X = h(a, g(f(a), f(f(a))), W)$, $U = a$, $V = g(f(a), f(f(a)))$

(c) `p(h(f(X),a),X,h(f(f(X)),f(f(X)))) eq p(h(V,a),U,h(W,f(V)))`
gives $X = U$, $V = f(U)$, $W = f(f(U))$

(d) `p(g(X),h(a,X),h(Y,Y)) eq p(U,h(a,g(V)),h(V,g(U)))`
gives false

Extend your implementation of rational trees to include the occur-check axiom of Clark's Equality Theory (CET). This means that infinite trees are now disallowed and should fail, an example of such a failing query: `X eq f(Y), Y eq f(X)`.

Submit the next exercise by e-mail to: amira.zaki@uni-ulm.de, latest by 06.02.2013 at 10:00.

Exercise 2. The theory of rational trees should define the (purely) syntactic inequality $\dot{\neq}$ between two terms :

irreflexivity	$\forall(x \dot{\neq} x \rightarrow \bot)$
symmetry	$\forall(x \dot{\neq} y \rightarrow y \dot{\neq} x)$
compatibility	$\forall(x_1 \dot{\neq} y_1 \vee \ldots \vee x_n \dot{\neq} y_n \rightarrow f(x_1,\ldots,x_n) \dot{\neq} f(y_1,\ldots,y_n))$
decomposition	$\forall(f(x_1,\ldots,x_n) \dot{\neq} f(y_1,\ldots,y_n) \rightarrow x_1 \dot{\neq} y_1 \vee \ldots \vee x_n \dot{\neq} y_n)$
distinctness	$\forall(\top \rightarrow f(x_1,\ldots,x_n) \dot{\neq} g(y_1,\ldots,y_m))$ if $f \neq g$ or $n \neq m$

Implement a CHR-constraint `X neq Y` that succeeds iff $CT \models$ `X`$\dot{\neq}$`Y`. The implementation should follow the one for syntactic equality. A satisfaction-complete implementation is not necessary. Implement the negated `same_args` by a CHR constraint `one_neq/2` that uses disjunction. The two arguments are lists of the same length and the constraint should succeed iff at least one pair of list-elements is unequal.

Queries:

(a) `X neq f(X)`
(b) `f(a,X) neq f(X,Y)`
(c) `f(g(X),a) neq f(Y,X)`

Constraint Programming

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Assignment #11

Constraint System $\Re$

Exercise 1 ($\Re$ as special RT solver)**.**
Modify the solver for rational trees (RTs) of Assignment 10:

```
idempotency   @ X eq Y \ X eq Y  <=> true.
reflexivity   @ X eq X           <=> true.
orientation   @ T eq X           <=> var(X), X@<T | X eq T.

decompostion  @ T1 eq T2         <=> nonvar(T1), nonvar(T2) | ...

confrontation @ X eq T1 \ X eq T2   <=> var(X), X @< T1, T1 @=< T2 | T1 eq T2.
```

as a polynomial equation solver by only changing the body of the `decomposition` rule to handle linear polynomial equations: The resulting equation `T1` minus `T2` is computed in normal form. An equation is in normal form if `X eq A + B*Y + ...` for variables, `X, Y, ...` and numbers `A, B, ....`

You can test your solver with queries like:

```
5+3*X eq 5+3*X.
Y eq 7+2*X+1*Z, Y eq -2+3*X, X eq 3.
Y eq 7+1*Z+2*X, Y eq -2+3*X, X eq 3.
```

Exercise 2 (Inequations Solver using Fourier Elimination)**.**
Implement an inequations solver for the `geq/2` constraint as discussed in the lecture: use Gaussian variable elimination for equations, use Fourier variable elimination and the bridge rule combining the variable elimination approaches for equations and inequations.

You can test your solver with queries like:

```
0+1*Y+1*X geq 0, 0+1*Y+(-1)*X geq 0.
3+2*X+4*Y geq 0, 12+4*X+2*Y eq 0.
```